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Method of Moments for Electromagnetic Modeling of Charge Distribution into Thin Wire

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Abstract

This paper represents the Method of Moments (MoM) for the evaluation of the linear charge density on finite straight segment of thin charged conducting wire. We assume that the charge density piecewise constant over the length and the electric potential are is one volt. The conducting structures are modeled by planar sub domains. The Method of Moments is employing pulse and delta as a basis and testing function, respectively, is used for analysis. The exact formulation for the matrix element is evaluated for sub domains, and we obtain a symmetric matrix of Toeplitz.

Keywords: *Methods of Moments, Charge density, Conducting Wire, Electric Potential, Toeplitz Matrix..*

1. Introduction

The Electromagnetic (EM) modeling and determinate of charge density systems has been the subject of extensive research in the last three decades. While in the past, design and distribution of charge may have been considered a secondary issue in overall system design, today it plays a critical role in spacecraft [1][2][9]. The accurate evaluation of charge distribution and capacitance of metallic structures is an important step in design of a high frequency integrated circuits. In this paper, it has been chosen the square shape of the sub domains because of its ability to conform easily to any geometrical surface or shape and at the same time to maintain simplicity of approach compared to the another shaped modeling. The MoM is based upon the transformation of an integral equation, into a matrix equation by employing expansion of the unknown in terms of known basis functions with unknown coefficients such as charge distribution is to be determined. Much of what the method of moments is used for analysis and design of antennas. As an antenna is thin, it is possible to view how it can be represented as a line.2. Electrostatic problems

Because electrostatic problems are relatively simple, the problem of finding the potential that is due to a given charge distribution is often considered. They provide a good context for introducing algorithms used to solve integral equations. In this section, we will consider an integral equation approach to solve for the electric charge distribution, once the electric potential is specified. The electric potential at point r due to an electric charge density q is given by the integral.

$$V(\mathrm{r}) = \frac{1}{4\pi\varepsilon_0}\int_{\substack{source\\ charge}} \frac{\mathrm{q}(r')}{R} dl' \tag{1}$$

If we know $V(r)$, we can obtain the electric potential everywhere. If we instead know the electric potential but not the charge density, (1) becomes an integral equation for an unknown charge density. We will now solve this problem numerically for a pair of practical examples, the charged wire and plate [1] [2][3].

Where $r'(x', y', z')$ denotes the source coordinates, $r(x, y, z)$ denotes the observation coordinates, dl' is the path of integration, and R is the distance from any point on the source to the observation point [1][3][4], which is generally represented by

$$R = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2} \tag{2}$$

Equation (1) is used to calculate the potentials that are due to any know line charge density. The charge distribution on most configurations of practical interest, complex geometries, is not usually known, even when the potential on the source is given. It is the nontrivial problem of determining the charge distribution, for a specified potential, that is to be solved here using an integral equation approach [1][3][4]. And our equation is a Fredholm integral equation, in general, these equations are written as

$$g(t) = \int_a^b K(x,t)\Phi(x)dx \qquad (3)$$

Where the functions $K(x,t)$ and $g(t)$ the limits a and b are known. The unknown function $\Phi(x)$ is to be determined; the function $K(x,t)$ is called the kernel of the equation. The moment method is a common numerical technique used in solving integral equations such as in equation 3 [1][2] [5].

3. Finite straight wire

Consider a straight conducting wire of radius a, and length L $(L >> a)$ located in free space along the y axis, as shown in figure 1. The wire is given a normalized constant electric potential of 1 V. Our goal is to determine the charge density λ along the wire using the moment of method. Once we determine λ, related field quantities can found [1][3] [4][5].

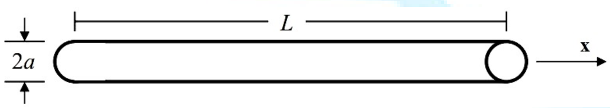


Fig. 1 Thin wire of constant potential.

Note that equation (1) is valid everywhere, including on the wire itself $(V_{wire} = 1V)$. Thus, choosing the observation along the wire axis $(x = z = 0)$ and representing the charge density on the surface of wire, at any point on the wire [1][4], equation (1) reduces to an integral equation of the form

$$1 = \frac{1}{4\pi\varepsilon}\int_0^L \frac{\lambda(x')}{R(x,x')}dl' \qquad 0 < x < L \qquad (4)$$

Where

$$R(x,x') = \sqrt{(x-x')^2 + \left[(x')^2 + (z')^2\right]} = \sqrt{(x-x')^2 + a^2} \qquad (5)$$

he observation point is chosen along the wire axis and the charge density is represented along the $\int_0^L f_n(x')dx' = f(x_1)\Delta x + \cdots f(x_N)\Delta x = \sum_{n=1}^{N} f(x_N)\Delta x$ surface of the wire to avoid $R(x,x') = 0$, which would introduce a singularity in the integrand of (4).

4. Thin wire segmentation

We search to transform equation (4) into linear system of equation, and applied the method of moment. Let subdivide the wire into N sub segments each of length $\Delta x = L/N$, as shown in figure 2 [1][3][4]. We assume that the charge density has a constant value and piecewise over the length of the wire [1][2][3].

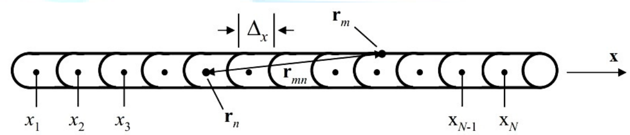


Fig. 2 Thin conducting wire held at a constant potential.

Mathematically, we write this as

$$q(x') = \sum_{n=1}^{N} \alpha_n f_n(x') \qquad (6)$$

Where a_n are unknown weighting coefficients, and $f_n(x')$ is a set of pulse functions that are constant on one segment but zero on all other segments

$$f_n(x') = \begin{cases} 0 & x' < (\mathrm{n}-1)\Delta x \\ 1 & (\mathrm{n}-1)\Delta x \le x' \le n\Delta x \\ 0 & x' > n\Delta x \end{cases} \qquad (7)$$

Since equation (4) applies for observation points everywhere one the wire, at a fixed point x_n known as the match point.

$$1 = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\lambda(x')}{|x-x'|}dx' \qquad (8)$$

The integration essentially finding in the area under a curve, if Δx is small, the integration of $f_n(x')$ over $0 < x < L$ is given by

(9)

Where the interval L has been divided into N units of each length Δx. And the wire are divided into N segments of equal length Δx as shown in figure 2, equation (8) becomes

$$4\pi\varepsilon_0 = \frac{\lambda_1 \Delta x}{|x_n - x_1|} + \frac{\lambda_2 \Delta x}{|x_n - x_2|} + \cdots + \frac{\lambda_N \Delta x}{|x_n - x_N|} \quad (10)$$

Where $\Delta x = L/N$ and we assuming in equation 10 is that the unknown charge density λ_n on the nth segment is constant. In equation (10) we have unknown constants $\lambda_1, \lambda_2, \ldots, \lambda_N$. Equation (10) must hold at all points on the wire, we obtain N similar equations by choosing N match points at $x_1, x_2, \ldots, x_k, \ldots x_N$ on the wire. Thus we obtain

$$\begin{aligned} 4\pi\varepsilon_0 &= \frac{\lambda_1 \Delta x}{|x_1 - x_1|} + \frac{\lambda_2 \Delta x}{|x_1 - x_2|} + \cdots + \frac{\lambda_N \Delta x}{|x_1 - x_N|} \\ 4\pi\varepsilon_0 &= \frac{\lambda_1 \Delta x}{|x_2 - x_1|} + \frac{\lambda_2 \Delta x}{|x_2 - x_2|} + \cdots + \frac{\lambda_N \Delta x}{|x_2 - x_N|} \\ \vdots \quad & \quad \vdots \qquad\quad \vdots \qquad\qquad \vdots \\ 4\pi\varepsilon_0 &= \frac{\lambda_1 \Delta x}{|x_N - x_1|} + \frac{\lambda_2 \Delta x}{|x_N - x_2|} + \cdots + \frac{\lambda_N \Delta x}{|x_N - x_N|} \end{aligned} \quad (11)$$

5. Matrix of charge density

We are guaranteed to find a unique solution for all values of the equation (11), we may therefore express the above system as a matrix vector equation with $Za = b$. And the match points $x_1, x_2, \ldots, x_k, \ldots x_N$ are placed at the center of each segment [5]. Equation (11) can be written as

$$[V_m] = [Z_{mn}][\lambda_n] \quad (12)$$

Where each Z_{mn} term is equal to

$$Z_{mn} = \int_0^L \frac{f_n(x')}{\sqrt{(x_m - x')^2 + a^2}} dx' = \int_{(n-1)\Delta x}^{n\Delta x} \frac{1}{\sqrt{(x_m - x')^2 + a^2}} dx' \quad (13)$$

With

$$[V_m] = [4\pi\varepsilon] \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

(14)

$$[Z_m] = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \ldots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} & \ldots & Z_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & Z_{N3} & & Z_{NN} \end{bmatrix}$$

(15)

$$Z_m = \frac{\Delta x}{|x_m - x_n|} \qquad m \neq n \quad (16)$$

$$[\lambda] = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N \end{bmatrix} \quad (17)$$

In equation (12), $[\lambda]$ is the matrix whose elements are unknown. We can determine $[\lambda]$ from equation (12) using Cramer's

$$[\lambda] = [Z]^{-1}[V]$$

rule, matrix inversion, or Gaussian elimination technique, using matrix inversion.

(18)

Where $[Z]^{-1}$ is the inverse of matrix $[Z]$, in evaluating the diagonal elements of matrix $[Z]$ in equation (11) or (15), caution must be exercised. Since the wire is conducting, a surface charge density σ_s is expected over the wire surface. Hence at the center of each segment

$$V(center) = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_{-\Delta x/2}^{\Delta x/2}\frac{\sigma_s ds}{\sqrt{(x^2+a^2)}}$$

$$= \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\int_{-\Delta x/2}^{\Delta x/2}\frac{\sigma_s a d\phi dx}{\sqrt{(x^2+a^2)}} \qquad (19)$$

$$= \frac{2a\sigma_s}{4\varepsilon_0}\log\left(\frac{\Delta x/2+\left[\sqrt{(\Delta x/2)^2+a^2}\right]}{-\Delta x/2+\left[\sqrt{(\Delta x/2)^2+a^2}\right]}\right)$$

Assuming Δ □ a

$$V(center) = \frac{2\pi a\sigma_s}{4\pi\varepsilon_0}2\log\left(\frac{\Delta x}{a}\right)$$

$$= \frac{2\sigma_s}{4\pi\varepsilon_0}\log\left(\frac{\Delta x}{a}\right) \qquad (20)$$

Where $\sigma_L = 2\pi a\sigma_s$. Thus, the self terms m=n are

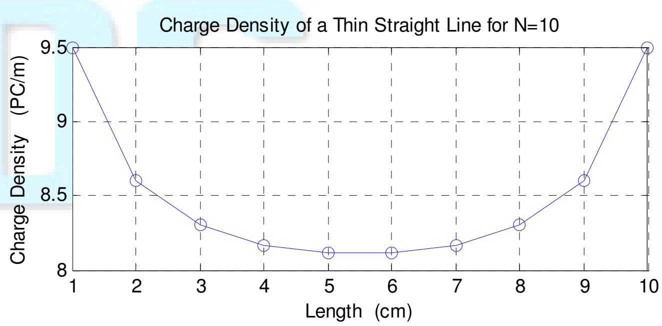
$$Z_{nn} = 2\log\left(\frac{\Delta x}{a}\right) \qquad (21)$$

Equation (12) now becomes

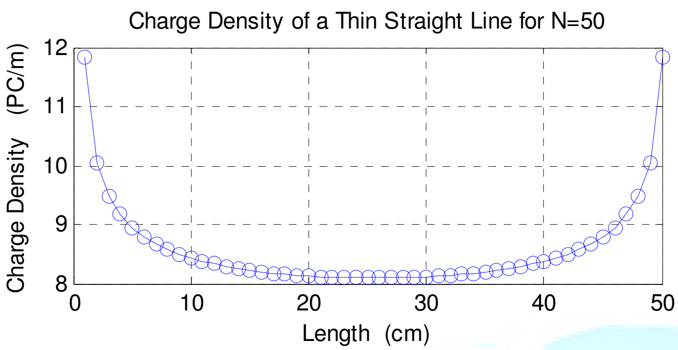
$$\begin{bmatrix} 2\log\left(\frac{\Delta x}{a}\right) & \frac{\Delta x}{|x_1-x_2|} & \cdots & \frac{\Delta x}{|x_1-x_N|} \\ \frac{\Delta x}{|x_2-x_1|} & 2\log\left(\frac{\Delta x}{a}\right) & \cdots & \frac{\Delta x}{|x_2-x_N|} \\ \vdots & & & \vdots \\ \frac{\Delta x}{|x_N-x_1|} & \frac{\Delta x}{|x_N-x_2|} & & 2\log\left(\frac{\Delta x}{a}\right) \end{bmatrix}\begin{bmatrix}\lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_N\end{bmatrix} = 4\pi\varepsilon_0\begin{bmatrix}1 \\ 1 \\ \vdots \\ 1\end{bmatrix} \qquad (22)$$

6. Result

Consider a thin conductive wire with length $L = 1m$ and radius, using equation (12) and with variation of N ($\Delta x = L/N$), a Matlab code can be developed, the plot is shown in figure 3. It should be expected that a smaller value of N would give a less accurate result and larger value of N would yield a more accurate result. However, if N is too large, we may have the computation problem of inverting the square matrix $[Z]^{-1}$. The capacity of the computing facilities at our disposal can limit the accuracy of the numerical experiment.



(a)



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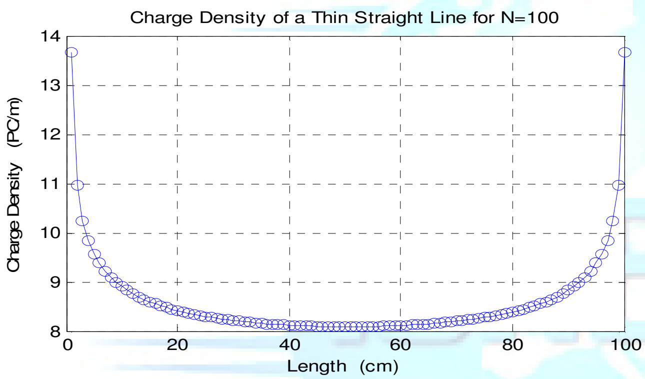

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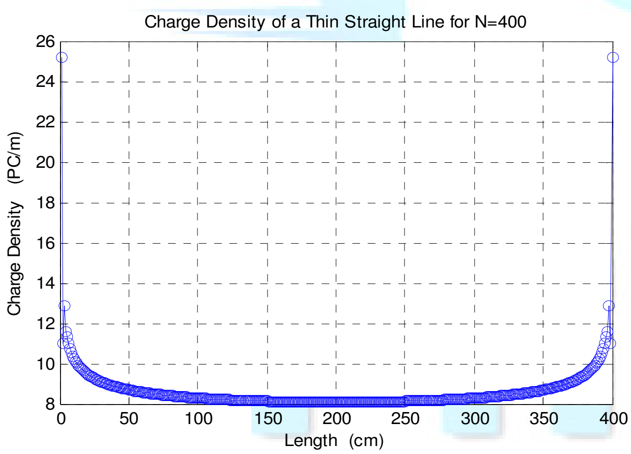
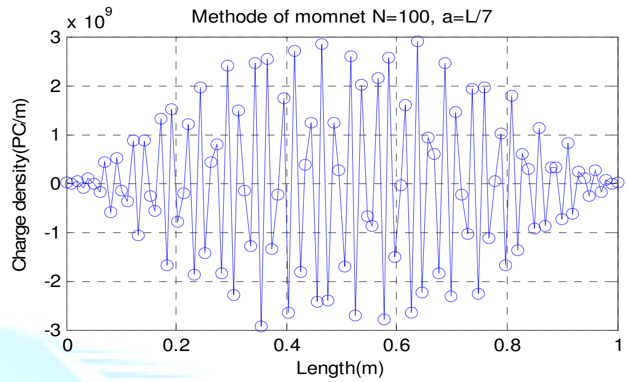
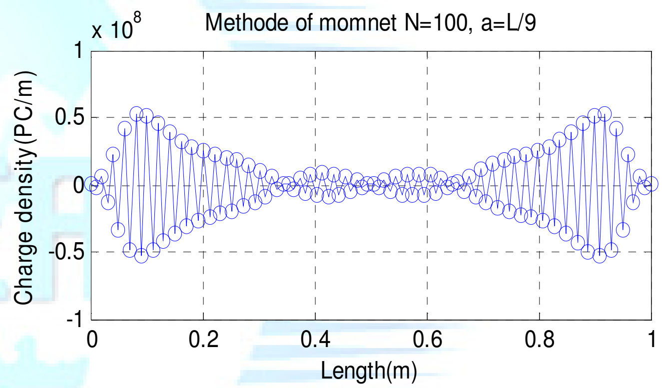


(d)

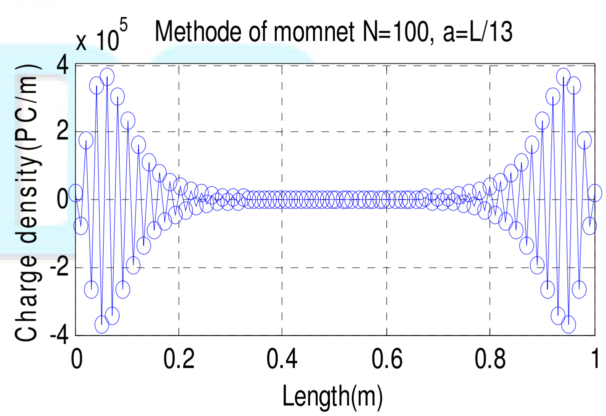
Fig. 3 Charge distribution on 1m straight wire at 1V
(a) N=10. (b) N=50. (c) N=100, (d) N=400



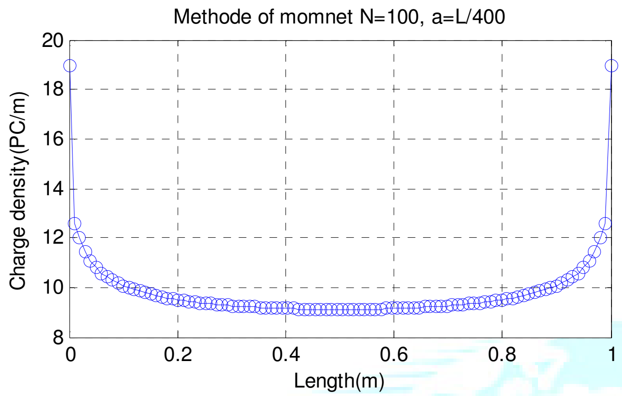
(e)



(f)



(g)



(h)

Fig. 3 Charge distribution into wire with variation of radius.
(e) a=L/7. (f) a=L/9. (g) a=L/13. (h) a=L/400

7. Conclusion

In all results, we show the computed charge density on the wire using 10, 50,100 and 400 segments, respectively. The representation of the charge at the lower level of discretization is somewhat crude, as expected. The increase to 400 unknowns greatly increases the fidelity of the result. Using the computed charge density, we then compute the potential at 100 points along the wire. The potential using 10 charge segments has a voltage near the expected value of 1V; it is not of constant value, especially near the ends of the wire. Our shows the potential obtained using 400 charge segments. The voltage is now nearly constant across the entire wire, except at the endpoints. Because we used a uniform segment size for the wire, the charge density tends to be somewhat oversampled in the middle of the wire and under sampled near the ends. As a result, the variation of the charge near the ends of the wire is not represented as accurately as in the center, and the computed voltage tends to diverge from the true value. Realistic shapes have irregular surface features such as cracks, gaps and corners that give rise to a more rapid variation in the solution at those points. In an attempt to increase accuracy, it is often advantageous to employ a denser level of discretization in the areas we expect the most variation.

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