

# Energy Metering Of Surge Load Based On ESPRIT Method

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## Abstract

More and more impact loads arising in power systems, they have the characteristics of fast and random power fluctuation, the voltage flicker and waveform distortion. Hence, the traditional measurement methods are no longer applicable. The Esprit harmonic analysis method is proposed in this paper, which can determine the harmonic components in a relatively short period, provide the voltage and current harmonic parameters accurately. Then, the active power and active electric energy can be calculated. In contrast with the traditional Hanning windowed interpolation FFT method, simulation and experimental results show that: the active energy error and the signal reconstruction RMSE of ESPRIT algorithm have greater advantages than the traditional method. Therefore, the proposed method can accurately detect the harmonic parameters in a short time window, and is especially suitable for electric energy metering of surge loads.

**Keywords:** Surge load; Active electric energy measurement; Esprit algorithm; Hanning windowed interpolation FFT method.

## Introduction

Energy Metering directly affects the equity trading between power generation, transmission, distribution, and client. However, with the increase of large amount of new energy, nonlinear and surge loads, the harmonic of the power network is much more complex, which leads to the reduction of measurement accuracy [1]. In order to adapt to the surge load of energy

measurement, it is necessary to study the harmonic analysis method for shock signal.

Nowadays, the harmonic analysis method which is widely used in engineering is the Hanning windowed interpolation FFT method. By means of window function and interpolation algorithm, the spectrum leakage and fence effect can be reduced effectively. But its frequency resolution depends on the length of the window, so it is not feasible to analyze the shock signal in theory [2,3]. In order to realize the spectrum analysis of short data, this paper uses modern spectrum analysis method, that is, to make full use of the difference between the data and the statistical characteristics of noise, to achieve high resolution and anti-noise in short time [4].

Common modern spectrum analysis methods are such as AR method, Prony method, Pisarenko method, MUSIC method, artificial neural network method, subspace method, etc. [5]. Among them, estimation of Signal Parameters via Rotational Invariance Techniques, namely ESPRIT method which is based on subspace decomposition, has been gradually applied to the power system harmonic analysis [6,7]. It can give a high precision frequency, phase, amplitude and attenuation parameters of the signal, no spectral peak search and have better anti-noise performance.

Therefore, the paper proposes to use ESPRIT algorithm to analyze the harmonics of the short data, and then realize the

energy measurement of the impact load. Specific simulation and measurement examples will be given in the following, and the harmonic analysis performance of the method is investigated in terms of waveform reconstruction error. The feasibility of the method which is applied to the impact load energy metering is investigated in terms of the active energy calculation error

## 1. Principle of harmonic analysis by ESPRIT method

Model of the noisy signal is as follows:

$$x(n) = \sum_{k=1}^K a_k e^{-\alpha_k n T_s} \cos(2\pi f_k n T_s + \phi_k) + w_r(n)$$

$$= \text{Re} \left( \sum_{k=1}^K s_k z_k^n + w(n) \right) = \text{Re}(\tilde{x}(n)) \quad (1)$$

Wherein  $s_k = a_k e^{-j\phi_k}$ ,  $z_k = e^{(-\alpha_k + j2\pi f_k)T_s}$ ;  $\alpha_k$ ,  $a_k$ ,  $f_k$ ,  $\phi_k$  are the  $k$ th harmonic attenuation, amplitude, frequency and phase parameters;  $T_s$  is sampling interval;  $w(n)$  is white gaussian noise. Now, the time window signal for the length of M is as follows:

$$\mathbf{x}(n) = [x(n) \quad x(n+1) \quad \dots \quad x(n+M-1)]^T$$

Vectors and matrices are defined as follows:

$$\mathbf{v}_M(n) = [1 \quad z \quad \dots \quad z^{M-1}]^T$$

$$\mathbf{s} = [s_1 \quad s_2 \quad \dots \quad s_K]^T$$

$$\mathbf{V}_M(n) = [\mathbf{v}_M(z_1) \quad \mathbf{v}_M(z_2) \quad \dots \quad \mathbf{v}_M(z_K)]$$

$$\Phi = \text{diag}(z_1, z_2, \dots, z_K)$$

$$\mathbf{w}(n) = [w(n) \quad w(n+1) \quad \dots \quad w(n+M-1)]^T \quad \text{There is}$$

$$\mathbf{x}(n) = \mathbf{V}_M \Phi^n \mathbf{s} + \mathbf{w}(n) = \mathbf{S}(n) + \mathbf{w}(n) \quad (2)$$

Where  $\mathbf{S}(n) = \mathbf{V}_M \Phi^n \mathbf{s}$

Rewriting A and B are as follows:

$$\mathbf{S}(n) = \begin{bmatrix} \mathbf{S}_1 \\ - \\ \text{last row} \end{bmatrix} = \begin{bmatrix} \text{first row} \\ - \\ \mathbf{S}_2 \end{bmatrix} \quad (3)$$

$$\mathbf{V}_M(n) = \begin{bmatrix} \mathbf{V}_1 \\ - \\ \text{last row} \end{bmatrix} = \begin{bmatrix} \text{first row} \\ - \\ \mathbf{V}_2 \end{bmatrix} \quad (4)$$

Obviously, we have

$$\mathbf{S}_1(n) = \mathbf{V}_1 \Phi^n \mathbf{s}, \quad \mathbf{S}_2(n) = \mathbf{V}_2 \Phi^n \mathbf{s},$$

Hence,

$$\mathbf{V}_2 = \mathbf{V}_1 \Phi \quad (5)$$

The specific implementation of the algorithm is constructing data matrix (6) by  $x(0), x(1), \dots, x(N-1), \dots, x(M+N-2)$

$$\mathbf{X} = \begin{bmatrix} x(0) & x(1) & \dots & x(M-1) \\ x(1) & x(2) & \dots & x(M) \\ \vdots & \vdots & \ddots & \vdots \\ x(N-1) & x(N-2) & \dots & x(N+M-2) \end{bmatrix} \quad (6)$$

Then the singular value is decomposed into  $\mathbf{X} = \mathbf{L}\mathbf{\Sigma}\mathbf{U}^H$ .

The right feature vector  $\mathbf{U}$  of  $\mathbf{X}$  contains the signal and noise information. It can be decomposed into signal subspace  $\mathbf{U}_s$  and noise subspace  $\mathbf{U}_n$ , namely  $\mathbf{U} = [\mathbf{U}_s | \mathbf{U}_n]$ .

$\mathbf{U}_s$  represents the first  $2K$  singular value corresponding to the column vector, thus,  $\mathbf{U}_s$  and  $\mathbf{V}_M$  expand the same signal subspace, there must be the following transformation:

$$\mathbf{V}_M = \mathbf{U}_s \mathbf{T}$$

Similarly,  $\mathbf{U}_s$  can be rewritten as:

$$\mathbf{U}_s = \begin{bmatrix} \mathbf{U}_1 \\ - \\ \text{last row} \end{bmatrix} = \begin{bmatrix} \text{first row} \\ - \\ \mathbf{U}_2 \end{bmatrix}$$

Accordingly,  $\mathbf{V}_1 = \mathbf{U}_1 \mathbf{T}$ ,  $\mathbf{V}_2 = \mathbf{U}_2 \mathbf{T}$

It shows:  $\mathbf{V}_2 = \mathbf{V}_1 \Phi \Leftrightarrow \mathbf{U}_2 \mathbf{T} = \mathbf{U}_1 \mathbf{T} \Phi$

$$\mathbf{U}_2 = \mathbf{U}_1 \mathbf{T} \Phi \mathbf{T}^{-1} = \mathbf{U}_1 \Psi \Rightarrow \Psi = \mathbf{T} \Phi \mathbf{T}^{-1}$$

It can be known that  $\Psi \sim \Phi$ ,  $z_k$  which is the eigenvalue of  $\Phi$  can be estimated by  $\lambda_k$  which is the eigenvalue of  $\psi$ .

$$U_2 = U_1 \Psi \Rightarrow \Psi = (U_1^H U_1)^{-1} (U_1^H U_2)$$

After calculating  $\psi$  and obtaining its eigenvalues, it can be estimated  $z_k, z_k = e^{(-\alpha_k + j2\pi f_k)T_s}$ , and then get

$$f_k \approx \frac{\text{angle}(\lambda_k)}{2\pi T_s}, \alpha_k \approx -\frac{\ln(\lambda_k)}{T_s}$$

Then the least square method is applied to calculate the amplitude and phase parameters. The signal can be reconstructed by harmonic parameters and compared with the original signal. Finally, the active energy can be calculated by the definition of active energy.

## 2. Harmonic analysis and simulation

Formula (7) and (8) show the simulated voltage and current signal whose fundamental frequency is 49.8Hz, sampling frequency is 5000Hz, sampling points are 1000, gauss white noise of  $w$  is SNR=60dB, the attenuation coefficient of current signal is  $x_1 = 0.8e^{-8t} + 0.2$

$$u(t) = 100 \cos(\omega t + \frac{\pi}{4}) + 5 \cos(3\omega t + \frac{\pi}{5}) + 2 \cos(5\omega t + \frac{\pi}{3}) + w \quad (7)$$

$$i(t) = 2 + x_1 * \begin{pmatrix} 10 \cos(\omega t) + 0.3 \cos(2\omega t) + \cos(3\omega t) \\ + 0.1 \cos(4\omega t) + 0.5 \cos(5\omega t) \end{pmatrix} + w \quad (8)$$

The following three schemes are used for the harmonic analysis of current signal: (1) the Hanning windowed interpolation FFT method whose window length is 10 cycles; (2) the Hanning windowed interpolation FFT method whose window length is 2 cycles; (3) ESPRIT method whose window length is 2cycles.

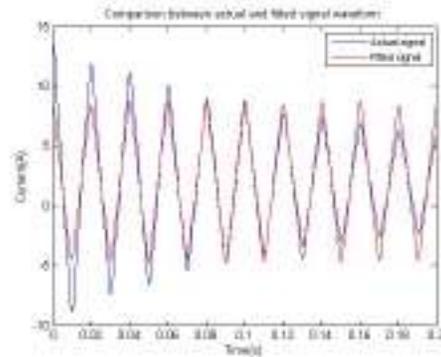
Then, performance evaluation is carried out, including: (1) the current waveform is reconstructed according to the results of harmonic analysis, and the RMSE (Root Mean

Square Error) is calculated; (2) the error of active energy calculation.

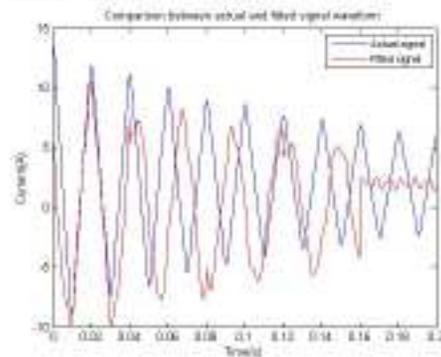
RMSE is defined as follows:

$$RMSE = \left[ \frac{1}{N} \sum_{j=1}^n (x(j) - \hat{x}(j))^2 \right]^{1/2} \quad (9)$$

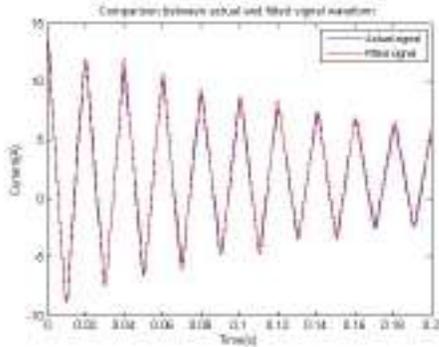
Where  $x(j)$  is the amplitude of the original signal at the sampling point  $j$ ;  $\hat{x}(j)$  is the magnitude at the sampling point  $j$  after restructuring;  $N$  is the length of the signal. The waveform of current signal reconstruction is shown in Figure 1:



(a) Hanning windowed interpolation FFT (1000 points) signal decomposition and reconstruction



(b) Hanning windowed interpolation FFT (200 points) signal decomposition and reconstruction



(c) ESPRIT algorithm signal decomposition and reconstruction

Fig.1 Comparison of fitting waveform and original current signal waveform

In addition, the reconstruction error RMSE and active energy error are shown in Table 1. The theoretical value of the active energy is 41.9489J.

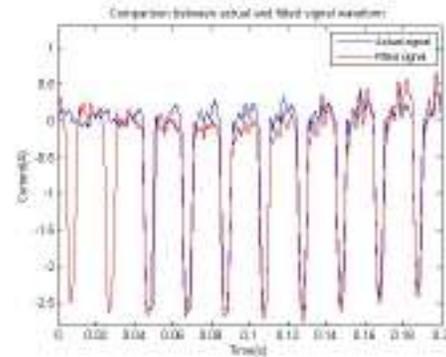
Table 1: Comparison of calculation results between FFT and ESPRIT

Algorithm	method		
	W(J)	W error	RMSE
FFT(10T)	41.1912	1.81%	1.3502
FFT(2T)	26.8990	35.88%	5.8980
ESPRIT(2T)	41.9008	0.11%	0.3982

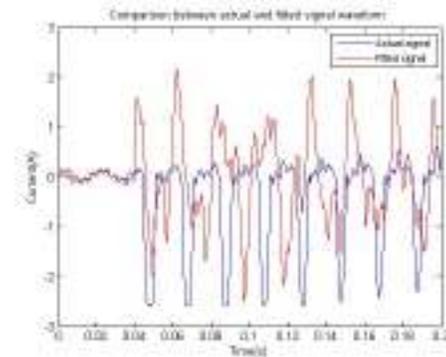
As shown in Figure 1 and Table 1: Hanning windowed interpolated FFT algorithm reconstruction RMSE and active energy error are larger. Mainly because of the low signal sampling point, resulting in a low frequency spectrum resolution, it cannot effectively analyze the harmonic of signal, and it indicates FFT cannot overcome the short period of harmonic parameters calculation. Although the reconstruction RMSE and active energy error of ten cycles of windowed FFT are smaller than the two cycles, the reconstructed signal from Figure 1 (a) is largely different from the original signal.

### 3. Example analysis of energy measurement of an electric railway

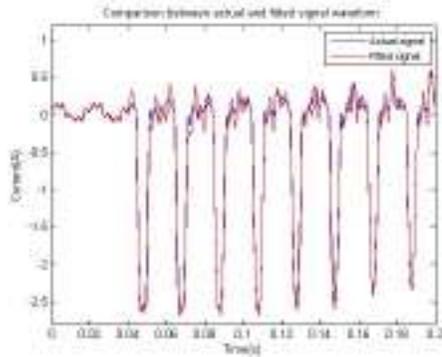
The current signal data collected by a section of electric railway is analyzed by using the above three methods. Sampling frequency is 5000Hz, sampling points are 1000, and sampling length is 0.2s. Comparison of the electric railway current signal and the fitting signal is shown in Figure 2, and the active energy is calculated by using the collected electric railway voltage signal, the current signal fitting RMSE and the active energy error are shown in Table 2. The theoretical value of the active energy is 6.8120J.



(a) Hanning windowed interpolation FFT (1000 points) signal decomposition and reconstruction



(b) Hanning windowed interpolation FFT (200 points) signal decomposition and reconstruction



(c) ESPRIT algorithm signal decomposition and reconstruction

Fig.2 Comparison of fitting waveform and electric railway current signal waveform

Table 2: Comparison of calculation results between FFT and ESPRIT method

Algorithm	W(J)	W error	RMSE
FFT(10T)	9.1383	34.19%	0.5265
FFT(2T)	3.6449	46.49%	1.1899
ESPRIT	6.8092	0.04%	0.1259

As shown in Figure 2 and Table 2: Using ESPRIT algorithm to calculate the harmonic parameters can be used to reconstruct the original current signal well. The signal reconstruction RMSE is 0.1259 and the calculated energy error is 0.04%, which are the minimum.

#### 4. Conclusions

In this paper, the ESPRIT method is applied to the high accuracy energy measurement in the case of harmonics and inter harmonics. The example results show: In the same sampling frequency and the length of the signal, The ESPRIT algorithm has better frequency resolution and anti-noise ability than FFT. Without pretreatment, it can directly eliminate the influence of noise and effectively identify the main harmonic and inter harmonic components

of the signal in a short data window. Effective reconstruction of the signal and the active energy of the load are obtained.

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