IJREAT International Journal of Research in Engineering & Advanced Technology, Volume 4, Issue3, June - July, 2016
ISSN: 2320 – 8791 (Impact Factor: 2.317)
www.ijreat.org

Adaptability Analysis of Nonlinear Load Reactive Measure Theory

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Abstract
At present, China has not yet developed reactive power measurement standards, the diversity of reactive power measurement standards lead to the conformity of the meters in market. Nowadays, the smart grid develops rapidly, and the extensive use of nonlinear loads causes the traditional measurement methods, which only consider fundamental reactive power, are no longer applicable. For these cases, this paper introduces various definitions of power, from the aspects of frequency domain, time domain, instantaneous reactive power, and general instantaneous reactive power theory, and points out strengths and weaknesses in specific conditions. At the same time, through the establishment of a fifth harmonic distortion of current, voltage model, the adaptability of the nonlinear load reactive power measurement theory has been simulated and analyzed. And it provides an efficient guidance for the determination of reactive power measurement standards.
Keywords: *Reactive Power Measurement Standards, Nonlinear Load, Definition of Power, Adaptability*

1. Introduction

In the modern power system, the power system structure, load structure, power supply type and characteristics are deeply changed. The key to the construction of smart grid is to strengthen the function of intelligent electric energy meter and improve the accuracy of measurement. The measuring principle of electric energy metering device has seriously restricted the further development of smart grid. Currently on the market, the results of the electric energy meter is not exactly same. As there is no uniform standard for reactive power measurement, it is not sure to judge who is the correct result of the measurement.

In the face of the increasingly complex non sinusoidal signal, the reactive power calculation is carried out in accordance with the traditional non sinusoidal reactive power calculation. Budeanu (1927), Fryze (1932) respectively studied nonsinusoidal sitiuation from the time domain and frequency domain of, and studied power and voltage and current of different definitions under the condition of nonsinusoidal periodic. They are used the most widely used two kinds of power definition so far. In addition, the instantaneous reactive power theory is established and generalized instantaneous reactive power theory is developed on the basis of instantaneous power.

2. Frequency Domain Reactive Power Theory

2.1 Budeanu Reactive Power (B Method)

In 1927, C.I.Budeanu put forward the theory of power under the condition of nonsinusoidal period [1]. He defined each of the frequency of the active power and reactive power as the total active power and reactive power, as shown in the formula 1-3:

Active Power: $$P = \sum_{n=1}^{\infty} P_n = \sum U_n I_n \cos\phi_n \qquad (1)$$

Reactive Power: $$Q_B = \sum Q_n = \sum U_n I_n \sin\phi \qquad (2)$$

Distorted Power: $$D_B = \left(S^2 - P^2 - Q_B^2\right)^{\frac{1}{2}} \qquad (3)$$

Budeanu definition has been written to the ANSI/IEEE standard, in 1941 by the income of "American electrical terminology standard definition" [2]. Later, many theories are developed on the basis of it. It still in the "IEEE Standard Dictionary" occupies a lot of space [3].

However, the Budeanu method was put forward in the limitations of the experimental environment at that time, there are some shortcomings:

1) The physical meaning of reactive power is not clear. When Q_n is nonzero, Q_B it may be zero; the B method

does not consider the reactive power of different frequency voltage and current [4].
2) It can not provide effective information for the system reactive power compensation.
3) Fu Liye transform must be taken before the calculation.
4) The introduction of distorted power does not have a physical meaning, but only for the balance power triangle.

2.2 Shepherd and Zakihani Reactive Power (S-Z)

Shepherd and Zakihani points out that the physical meaning of reactive power of Budeanu method is not clear. In 1973, they put forward the theory of full power to obtain the maximum value of the power factor [5]. They proposed the power definition as shown in the formula 4-6:
Active Full Power:

$$S_R^2 = \sum_{keG} U_n^2 \sum_{keG} I_m^2 \cos^2(\beta_n - \alpha_m) \tag{4}$$

Reactive Full Power:

$$S_X^2 = \sum_{keG} U_n^2 \sum_{keG} I_m^2 \sin^2(\beta_n - \alpha_m) \tag{5}$$

Distorted Full Power:

$$S_D^2 = \sum_{keG} U_n^2 \sum_{keR} I_m^2 + \sum_{keP} U_n^2 \sum_{keG} I_m^2 + \sum_{keP} U_n^2 \sum_{keR} I_m^2 \tag{6}$$

Among them, n , m is the harmonic number, β_n、α_m are the initial phase angle of voltage and current; G is a harmonic set of current and voltage at the same time; P is a harmonic set when the voltage harmonic only exits; R is a harmonic set when the current harmonic only exits.

The $S-Z$ method has the following characteristics [6]:
1) By compensating the reactive power, the power factor can be improved;
2) S_R of the physical meaning is not clear.
3) In the case of waveform distortion, this method calculates a large amount.

In 1973, on the basis of the S-Z method Sharon improved the reactive full power, defined as the formula 7 [7]:
Reactive Full Power:

$$S_X^2 = U_{rms}^2 \sum_{leG} I_l^2 \sin^2\left((\theta_u)_{(l)} - (\theta_i)_{(l)}\right) \tag{7}$$

Sharon reactive power definition solves the problem of current discontinuity by S-Z method in the consideration of reactive power compensation with capacitor inductance.

2.3 Emanuel Reactive Power

In 1974, Emanuel divided apparent power into active power and power compensation [8]. The definition is as shown in the formula 8-10:

Apparent Power:

$$S = UI = \sqrt{\sum_n U_n^2} \sqrt{\sum_n I_n^2} \tag{8}$$

Active Power:

$$P = \sum_{n=1}^{\infty} P_n = \sum U_n I_n \cos(\varphi_n) \tag{9}$$

Compensating Power:

$$P_G = \sqrt{S^2 - P^2} \tag{10}$$

Emanuel reactive power theory is helpful to the meter measurement and electric energy charge.

2.4 Czanecki Reactive Power

In 1983, Czaneckit put current and power into orthogonal decomposition and combing time domain with frequency domain analysis method, divided current into active current i_a , reactive current i_r , spread current i_s [9]. As shown in the formula 11-12:

$$\frac{\dot{I}_n}{\dot{U}_n} = G_n + j\beta_n , G_e = \frac{P}{U^2} \tag{11}$$

$$i_a = \sqrt{2} \operatorname{Re} \sum G_e \cdot U_n e^{jn\omega t}$$

$$i_s = \sqrt{2} \operatorname{Re} \sum (G_n - G_e) \cdot U_n e^{jn\omega t}$$

$$i_r = \sqrt{2} \operatorname{Re} \sum (j\beta_n) \cdot U_n e^{jn\omega t} \tag{12}$$

1) $i_a = \sqrt{2} \operatorname{Re} \sum G_e \cdot U_n e^{jn\omega t}$ is the active current i_p , and the Fryze active current has the same meaning;
2) $i_s = \sqrt{2} \operatorname{Re} \sum (G_n - G_e) \cdot U_n e^{jn\omega t}$ is the spread current, defined as $i - i_a - i_r$, that is the difference between i_q and i_r ;
3) $i_r = \sqrt{2} \operatorname{Re} \sum (jB_n) \cdot U_n e^{jn\omega t}$ is the reactive current, because of the difference between the phase of voltage and the phase of harmonic current[10]. Its power definition is as shown in the formula 13-16:
Apparent Power:

$$S = UI = \sqrt{\sum_{n=0}^{\infty} U_n^2}\sqrt{\sum_{n=0}^{\infty} I_n^2} \quad (13)$$

Active Power:

$$P = \sum_{n=1}^{\infty} P_n = \sum U_n I_n \cos(\varphi_n) \quad (14)$$

Reactive Power:

$$Q_r = UI_r = \sqrt{\sum_{n=0}^{\infty} U_n^2}\sqrt{\sum_{n=1}^{\infty} B_n^2 U_n^2} \quad (15)$$

Spreading Power:

$$D_s = \sqrt{S^2 - P^2 - Q_r^{\,2}} = UI_s \quad (16)$$

Czanecki reactive power definition is more detailed than the Fryze and S-Z method, and plays a great role in the study of harmonic and reactive power analysis [11]. The defect is: the final power of the fundamental active power in the apparent power is not easy to identify.

3. Time Domain Reactive Power Theory

3.1 Fryze Reactive Power

In 1932, Fryze analysed reactive power in time domain [12].He decomposed current into active current i_p and reactive current i_q according to whether in accordance with voltage waveform, the single-phase circuit is defined as shown in equation 17.

$$i_p = \frac{\frac{1}{T}\int_0^T uidt}{\frac{1}{T}\int_0^T u^2 dt} u = \frac{P}{U^2} u \quad (17)$$

$$i_q = i - i_p \quad (18)$$

Active Power:

$$P = UI_p = \sqrt{\frac{1}{T}\int_0^T u^2 \mathrm{d}t}\sqrt{\frac{1}{T}\int_0^T i_p^{\,2} \mathrm{d}t} = \frac{1}{T}\int_0^T ui\mathrm{d}t \quad (19)$$

Reactive Power:

$$Q_F = UI_q = \sqrt{S^2 - P^2} \quad (20)$$

Fryze definition of reactive power strongly influenced the apparent power theoryt in second half of the twentieth Century [13-15]. The advantages of Fryze time domain definition is eliminating the trouble of Fu Liye transformation, so we can directly get three kinds of power. But in fact Q_F is not reactive, it is a new full power, so it does not have a clear physical meaning. It can not provide information to improve the power factor.

3.2 Kusters and Moore Reactive Power

In 1980, Kusters and Moore used the calculus method decompose the current into active current i_p, inductance of rereactive current i_{ql} (capacitive re reactive current i_{qc}) and residual inductive reactive current i_{qlr} [16] (residual capacity of the i_{qcr} without reactive current), as shown in equation 21:

$$i_{ql} = \frac{\frac{1}{T}\int i\overline{u}dt}{\frac{1}{T}\int \overline{u}^2 dt}\overline{u} \text{ or } i_{qc} = \frac{\frac{1}{T}\int i\dot{u}dt}{\frac{1}{T}\int \dot{u}^2 dt}\dot{u} \quad (21)$$

In the above formula, $\dot{u}$, $\overline{u}$ respectively express of the derivative and integral voltage u ;

We can find reactive power calculation formula in the formula 22-26:

Apparent Power:

$$S = UI = \sqrt{\frac{1}{T}\int_0^T u^2 \mathrm{d}t}\sqrt{\frac{1}{T}\int_0^T i^2 \mathrm{d}t} \quad (22)$$

Active Power:

$$P = UI_p = \sqrt{\frac{1}{T}\int_0^T u^2 \mathrm{d}t}\sqrt{\frac{1}{T}\int_0^T i_p^{\,2} \mathrm{d}t} = \frac{1}{T}\int_0^T ui\mathrm{d}t \quad (23)$$

Inductive Reactive Power:

$$Q_L = UI_{ql} = U\frac{\frac{1}{T}\int i\overline{u}dt}{\frac{1}{T}\int \overline{u}^2 dt}\sqrt{\frac{1}{T}\int \overline{u}^2 dt} = \left(\frac{1}{T}\int i\overline{u}dt\right)\frac{U}{\overline{U}} \quad (24)$$

Capacitive Reactive Power:

$$Q_c = UI_{cl} = U\frac{\frac{1}{T}\int i\dot{u}dt}{\frac{1}{T}\int \dot{u}^2 dt}\sqrt{\frac{1}{T}\int \dot{u}^2 dt} = \left(\frac{1}{T}\int i\dot{u}dt\right)\frac{U}{\dot{U}} \quad (25)$$

Residual Power:

$$D_r = \sqrt{S^2 - P^2 - Q_r^{\ 2}} = UI_{qlr}(I_{qcr}) \tag{26}$$

Kusters and Moore the definition of reactive power has been recognized by the International Electrical Commission, which can be used to approximate the reactive current compensation. All kinds of components can be measured by the instrument, but in the process of dealing with the inter harmonics and random signals, the obstacles will be encountered.

In 1980, C.H.Page made a correction and extension to it. He divided the current into three parts: active current i_p, reactive current i_{qu} and distorted current i_d. The formula was calculated as 27:

$$i_p = \frac{\frac{1}{T}\int_0^T iudt}{\frac{1}{T}\int_0^T u^2 dt} u = \frac{P}{U^2} u$$

$$i_q = i - i_p = i_{qu} + i_d$$

$$i_{qu} = \frac{-Q_H}{U^2} H\{u(t)\} \tag{27}$$

Its power definition is shown in the formula 28-31:

Apparent Power:

$$S = UI = \sqrt{\frac{1}{T}\int_0^T u^2 \mathrm{d}t}\sqrt{\frac{1}{T}\int_0^T i^2 \mathrm{d}t} \tag{28}$$

Active Power:

$$P = UI_p = \sqrt{\frac{1}{T}\int_0^T u^2 \mathrm{d}t}\sqrt{\frac{1}{T}\int_0^T i_p^{\ 2} \mathrm{d}t} = \frac{1}{T}\int_0^T ui\mathrm{d}t \tag{29}$$

Reactive Power:

$$Q_H = \pm UI_{qu} = (u(t), H\{i(t)\}) = (-i(t), H\{u(t)\}) \tag{30}$$

Distorted power:

$$Q_D = UI_d = \sqrt{S^2 - P^2 - Q_H^2} \tag{31}$$

The upper H represents Hilbert transformation. This definition is applicable to all periodic waveforms and is no longer limited to the direction of reactive power, which can be measured by the actual engineering.

4 Instantaneous Reactive Power Theory

Under the condition of nonsinusoidal power research, experts note that by two constant active power and reactive power, it is difficult to achieve non sinusoidal power analysis, and put forward the instantaneous active power and instantaneous reactive power concept.

In 1983, H.AKagi introduced instantaneous reactive power and instantaneous reactive current concept for the purpose of reactive power compensation.Design three-phase circuit instantaneous voltage and instantaneous current respectively as u_A, u_B, u_C, i_A, i_B, i_C, then the they are transformed to α-β coordinates, two mutually orthogonal, finally get two-phase transient voltage u_α, u_β and i_α, i_β, process is as shown in equation.:

$$\begin{bmatrix} u_\alpha \\ u_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix}$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} \tag{32}$$

The instantaneous active and reactive power are respectively:

$$p = u_\alpha i_\alpha + u_\beta i_\beta, \quad q = u_\alpha i_\beta - u_\beta i_\alpha \tag{33}$$

The instantaneous reactive power theory is characterized by:

1) Get the compensation current by Park transform and inverse transform, so that reduce the energy transfer process loss. It can be use in harmonic and reactive power instantaneous detection.
2) The three-phase energy is optimized with the minimum instantaneous transmission energy, and the angle of the cycle is not the least.
3) The theory is only applicable to three-phase three wire circuit, and the zero sequence component can not be treated well.
4) In the case of nonsinusoidal and asymmetric loads, the definition of the theory has no clear physical meaning.

Aki Wentai's instantaneous reactive power theory, which has played a considerable role in promoting the development of the theory of instantaneous power, has played a considerable role in promoting the development of the theory of instantaneous power.

5 Universal Instantaneous Power Theory

In the nonsinusoidal circuit, the reactive power is the same with the sine circuit, which indicates the energy of the reciprocating oscillation. Oscillation of excess energy increases transmission line energy loss. In the case of meeting the load demand, the minimum line loss is compensated.The output current of the power supply is the active current, and the output current of the compensation device is the reactive current. When active power consumption is constant , the reactive power and the line loss is proportional . Transmission loss is minimal when the power supply is the lowest. In the three-phase three wire circuit, to reduce the energy loss of the transmission line, the model of the active current is defined as the formula 34-36:

$$\mathrm{Min}\int_0^T (i_{pA}^2 + i_{pB}^2 + i_{pC}^2)dt \tag{34}$$

$$\int_0^T (u_A i_{pA} + u_B i_{pB} + u_C i_{pC})dt = \int_0^T (u_A i_A + u_B i_B + u_C i_C)dt \tag{35}$$

$$i_{pA} + i_{pB} + i_{pC} = 0 \tag{36}$$

We can find the definition of reactive current and three-phase instantaneous active power, instantaneous reactive power in the formula 37-38:

$$\begin{bmatrix} i_{qA} \\ i_{qB} \\ i_{qC} \end{bmatrix} = \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} - \begin{bmatrix} i_{pA} \\ i_{pB} \\ i_{pC} \end{bmatrix} \tag{37}$$

$$p = \begin{bmatrix} p_A \\ p_B \\ p_C \end{bmatrix} = \begin{bmatrix} u_A i_{pA} \\ u_B i_{pB} \\ u_C i_{pC} \end{bmatrix}, \quad q = \begin{bmatrix} q_A \\ q_B \\ q_C \end{bmatrix} = \begin{bmatrix} u_A i_{qA} \\ u_B i_{qB} \\ u_C i_{qC} \end{bmatrix} \tag{38}$$

In universal definition of instantaneous power model, objective function and constraint conditions are the integral form of a cycle. When the three-phase voltage and current is periodic, universal definition of instantaneous power can make full use of the periodic quantity of information. In terms of it, the universal definition of instantaneous power is more perfect than AKagi instantaneous power definition.

6 Reactive Power Theory Simulation and Comparison

According to the frequency domain and time domain reactive power measurement model, the actual data are given respectively. The simulation results are compared with the advantages and disadvantages of each model. The selected voltage and current models are shown in Table 1:

Table.1 Voltage and current data models

Harmonic	Voltage	electric current	Voltage phase	Current phase
DC component	20	2	0	0
Fundamental	220	12	0	10
Three harmonic	35	3	30	20
Five harmonic	12	1	150	0

According to the data given in Table 1, with the time domain and frequency domain of each reactive power measurement model, the results obtained are shown in table 2:

Table.2 Calculation results

Theory		P	Q	D
Frequency domain	Bedeanu	2732.9	-434.1981	519.0418
	S-Z	2771.8	493.9612	
	Sharon	2771.8	493.9612	
	Emanuel	2732.9	676.7070	
	Czanecki	2732.9	493.9612	462.5308
	General reactive power	2732.9	676.7070	
Time domain	Fryze	2732.9	676.7070	
	Kusters and moore	2732.9	QL=-451.1535 QC=-373.7320	
	C.H.page	2732.9	±676.7070	

According to the results of the operation of the data, power line diagram 1 can be formated, the difference between the power theory can be more obvious looked out.

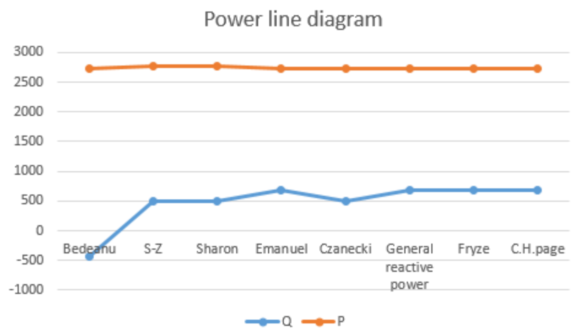

Fig. 1 Power line diagram

From the table 2 and figure 1, we can see that the power theory can be roughly divided into three categories:one is Budeanu, the traditional reactive power theory, the second is Czanecki, Shepherd, Sharon theory; the third is Fryze, C.H.page power definition theory.

Budeanu's reactive power theory in a certain extent is similar to conventional power theory, in practice reactive power measure, reactive power compensation still has a lot of deficiencies; Shepherd and Emanuel definition of reactive power has a great help in the measuring instrument and reactive power management, fees, etc.; Czarnecki reactive theory can be used for harmonic and reactive power compensation, which is convenient for actual measurements. But there is no obvious physical meaning; Kusters and Moore's reactive power theory is mainly used for reactive current compensation.

7. Conclusions

Firstly, the paper introduces the main reactive power measurement model in frequency domain and time domain, and gives the calculation formula of the two aspects. According to the time development clue, it is not difficult to find the transfer relationship between theory. The advantages and disadvantages of each theory are analyzed, each theory is based on the original basis to solve the problems left, but more or less bring into the new problem. Then the instantaneous reactive power theory and the universal instantaneous power theory of three-phase circuit reactive power calculation are introduced.

Based on the five harmonic distortion current and voltage model, simulation and comparison are made on the adaptability of the theory of reactive power measurement of nonlinear load. According to the results, we found that the definition of power can be roughly divided into three categories: a class is budeanu's traditional reactive power theory, the second is Czanecki shepherd, and Sharon theory; third class is the Fryze, C.H.page definition of power theory.

The definition of Budeanu retains the information of each spectral component, but the computation is very large, can not be measured in real time as well as the compensation. Shepherd definition of the full power is designed to compensate for some of the components so that the maximum power factor.It has a good use value in the reactive power compensation, but it lacks physical meaning in terms of active power. When the load is nonlinear or applied nonsinusoidal voltage , the result of the calculation deviats from the traditional meaning one. Fryze power definition decomposes current into two mutually orthogonal parts to realize the current compensation, suitable for harmonic and reactive power compensation and control. But it also lost each frequency component of the data. Research for signal decomposition, harmonic sources in power grid determination based on signal spectrum to identify fails.

At present, the definition of reactive power is varied, and no one is defined by the majority of electrical authors, so the definition of nonsinusoidal reactive power needs more research to do. The next study on reactive power theory is to further improve the existing universal definition of reactive power or propose new definition of reactive power, which is convenient for measurement, and convenient to control, also has clear physical meaning.

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